Calculus AB

Brief Description of the Course

This course is intended for students who have a thorough knowledge of analytical geometry and elementary functions in addition to algebra, geometry, and trigonometry. This course is a rigorous treatment of the topics of elementary calculus. Algebraic, trigonometric, exponential, and logarithmic functions will be studied, with an emphasis on their properties and limits. Differential calculus will be studied and will include defining the derivative, taking the derivative of functions, and applying the derivative to related rates problems and maximum-minimum problems. The following topics of integral calculus will be studied: Antiderivatives and their applications; definite integrals and their applications, including area under the curve, velocity and acceleration, and volumes of solids of revolution; and techniques of integration.

Unit Information

I. Chapter P: Preparation for calculus.
   a) Graphs and models
   b) Linear models and rates of change
   c) Functions and their graphs
   d) Fitting models to data

II. Chapter 1 – Limits and their properties
   a) A preview of calculus
   b) Finding limits graphically and numerically
   c) Evaluating limits analytically
   d) Continuity and one-sided limits
   e) Infinite limits

Major Assignments/Assessments:

Activity 1: Approaching limits

This activity will provide students with various views of how a function behaves as the input approaches a particular value. Students will obtain the following skills:

a) The ability to produce and manipulate graphs and tables of values manually and with a graphing calculator.
b) Basic understanding of function language.
c) The ability to identify rational, exponential, and trigonometric functions

Given either a function, table, or graph students will state and explain the limit at particular values.

Activity 2: Is there a limit to what side you take?

This activity will explain one-sided limits. Students will obtain the following skills:

a) A working knowledge of piecewise functions including producing piecewise functions on a graphing calculator
b) The ability to manipulate graphs and tables of values manually and use the ZOOM feature on a graphing calculator

Given a function, students will be able to state and explain the limit at a particular value using graphical and numerical analysis

III. Chapter 2: Differentiation
   a) The derivative and the tangent line
   b) Basic differentiation rules and rates of change
   c) The product and quotient rules and higher order derivatives
   d) The chain rule
   e) Implicit differentiation
   f) Related rates

Major Assignments/Assessments
Activity 3: Local linearity, differentiability, and limits of difference quotients
In this activity, the local linearity of several functions at different points is discussed and explored. The concept of local linearity is linked to differentiability by examining one-sided limits of difference quotients of several functions numerically as h approaches 0.
Students can investigate the local linearity given a function and a point and then connect that notion with the functions differentiability at that point.

Lab: Discovering the Derivative
   a) Students will define the slope of a function at a point by zooming in on that point
   b) Students will develop the definition of the derivative of a function at a point by examining the slopes of secant lines
   c) Students will understand situations in which the derivative fails to exist

This lab encourages students to discover the limit definition of the derivative by looking at graphs of functions and secant lines that approach the tangent line to the graph at a point.

IV. Chapter 3: Applications of the Derivative
   a) Extrema on an interval
   b) Rolle's Theorem and the Mean Value Theorem
   c) Increasing and decreasing functions and the First Derivative Test
   d) Concavity and the Second Derivative Test
   e) Limits at infinity
   f) A summary of curve stretching
   g) Optimization problems
   h) Newton's method
   i) Differentials

Major Assignments/Assessments
Activity 4: Graphs of functions and their derivatives
In this activity, the concepts of increasing and decreasing function behavior are defined. This is followed by a graphical and symbolic exploration designed to show students how and why the derivative can be used as an indicator for this behavior. The concepts of local maxima and minima are then informally defined, followed by questions that allow students to uncover the ideas behind the first derivative test. Students should be able to state in written form how the derivative can help find local maxima and minima where differentiable.
Activity 5: Students will make freefall measurements by using an apparatus that provides the students the ability to graph 2-dimensional position of an object in free-fall in equal time intervals. The students will then graphically determine the position equation and use derivatives to determine both the velocity and acceleration equations for freefall.
Activity 6: Students will build "penny" boats, which are constructed out of a square (10 cm x 10 cm AL foil) and are designed to hold as many pennies as possible without sinking in a sink of water. Students will be required to maximize volume and determine the height (based on the

geometric shape) that accomplishes this. Graphing calculator plots and differentiation will be emphasized.

Chapter 4: Integration

a) Antiderivatives and indefinite integration
b) Area
c) Riemann sums definite integrals
d) The Fundamental Theorem of Integral Calculus
e) Integration by substitution
f) Numerical Integration

Major Activities/Assignments:

Activity 7: Approximating integrals with Riemann sums
In this activity, students will calculate and analyze Riemann sums. They will draw rectangles whose areas correspond to terms of Riemann sums and observe the convergence of left-handed and right-handed Riemann sums by using a graphing calculator to automate the production of Riemann sums with regular partitions having a large number of subintervals.
Activity 8: Students will determine distance and speed of an object under uniform acceleration. They will use graphing techniques and antiderivatives. A free-fall apparatus will be used.
Lab: Area Functions

a) Students will be introduced to the idea of area under a curve as a function
b) Students will extend this idea to accumulation functions
c) Students will make conjectures about the derivative of such functions

Activity 9: Introduction to slope fields
This activity is used as the students' first introduction to the idea of slope fields. Students will use a graphing calculator with an overhead interface to project grid points on a whitewall. Students will alternatively draw one of the line segments making up the slope field for a particular dy/dx. Students will be able to create their own slope fields for a given differential equation and grid points using pencil and paper.

V. Chapter 5: Integration
   a) The natural logarithmic function: differentiation
   b) The natural logarithmic function: integration
   c) Inverse functions
   d) Exponential functions: differentiation and integration
   e) Bases other than e and applications
   f) Differential equations: growth and decay
   g) Differential equations: separation of variables
   h) Inverse trigonometric functions: differentiation
   i) Inverse trigonometric functions: integration

VI. Chapter 6: Applications of Integration
   a) Area of a region between two curves
   b) Volume: disk method
   c) Volume: shell method
   d) Arc length and surface of revolution

Major Assignments/Assessments
Lab: Arc length

a) Students will develop the idea of approximating arc length by the sum of the lengths of straight line approximations to the curve
b) Students will compute arc length using an interval

Textbooks

Calculus with Analytical Geometry 7th edition by Ron Larson and Hostetler Robert, 2002. Publisher: Houghton-Mifflin Co.

Websites

URL: www.calculus-help.com/funstuff/phobe.html

This web site contains animations that explain the following concepts:
Ch. 1 – What is a limit; When does a limit exist; How to evaluate a limit; Limits at infinity; Continuity; The Intermediate Value Theorem
Ch. 2 – The difference quotient; The power rule; The product rule; The quotient rule; The chain rule